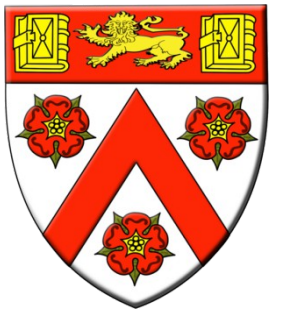

Mathematicians I have known

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http://www.maths.ed.ac.uk/~aar/atiyahpg

Trinity Mathematical Society
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The Michael and Lily Atiyah Portrait Gallery
James Clerk Maxwell Building, University of Edinburgh

The portraits of mathematicians displayed in this collection have been personally selected by us.

They have been chosen for many different reasons, but all have been involved in our mathematical lives in one way or another; many of the individual texts to the gallery portraits explain how they are related to us.

First, there are famous names from the past – starting with Archimedes – who have built the great edifice of mathematics which we inhabit. This early list could have been more numerous, but it has been restricted to those whose style is most appealing to us.

Next there are the many teachers, both in Edinburgh and in Cambridge, who taught us at various stages, and who directly influenced our careers.

The bulk of the portraits are those of our contemporaries, including some close collaborators and many Fields Medallists.

Lily has a special interest in women mathematicians: they are well represented, both past and present.

Finally we come to the next generation: our students. Of course, many of the categories overlap, with students later becoming collaborators and friends.

It was hardest to keep the overall number down to seventy, to fit the gallery constraints!

1) Classical

Leonhard Euler

Studied in Basel. Worked in St. Petersburg and Berlin. FRS.

His name appears in all branches of mathematics: Euler characteristic, Euler-Lagrange equations, Euler's constant. He gave the symbols to various important constants that we all use, such as e and π.

Euler's solution of the Königsberg bridge problem was the beginning of topology.

Euler introduced the famous infinite product that embodies the unique factorization of integers into prime factors, found the beautiful formula $\sum 1/n^2 = \pi^2/6$, and the fundamental relation $e^{i\theta} = \cos(\theta)+i\sin(\theta)$ between the exponential function and trigonometry.

Blind towards the end of his life, Euler continued to work until the very end – an encouragement to the older generation of the present time.

Niels Henrik Abel

His story of young genius, with a few years of creative life followed by an early death from tuberculosis, is iconic and is why the Abel Prize was founded in 2002.

His ideas on algebraic integrals were far in advance of his time, going well beyond the known theory of elliptic functions. The general view-point he adopted is at the foundation of modern mathematics. He was the first to show that the general equation of degree 5 is not soluble by radicals.

The brevity and euphony of his name has widened his appeal, with abelian functions, integrals, groups, varieties and categories. Having your name become an adjective – without a capital – is the mark of real fame, as exemplified earlier by the terms *euclidean* and *newtonian*.

Bernhard Riemann

A student of Gauss, Riemann worked in Göttingen, and was a foreign member of the Royal Society.

His name and work live on in Riemannian geometry, Riemann surfaces, the Riemann Hypothesis, the Riemann-Roch Theorem, the Riemann-Hilbert problem, and many more.

His foundational work on differential geometry was created as a sideline, to impress his doctoral examiners.

Unlike Euler his collected works take up only one modest volume, but each chapter opens a new door in mathematics.

Riemann died of tuberculosis at the height of his powers, aged just 40.

William Rowan Hamilton

At 21, he was appointed Royal Astronomer of Ireland, while still an undergraduate at Trinity College Dublin. Honorary FRSE.

He made his name early by predicting conical refraction, experimentally confirmed a short while later. Now famous for Hamiltonian mechanics, which became the paradigm for quantum mechanics a century later.

Hamilton is famous in mathematics for his discovery of the quaternions, carving the equations on Broom Bridge in Dublin where he had his brainwave in 1843. He opened the door to non-commutative algebra and, in physics, to the role of the group SU(2).

In one famous passage in an 1846 paper, Hamilton notes that a first order differential operator arising naturally in quaternions is the square root of the Laplace operator. He opines that this must have deep applications in physics. His forecast was fully justified since his operator was what Dirac rediscovered some 80 years later.

Solomon Lefschetz

Educated in France, moved to the United States in 1905. Professor in Kansas and Princeton. Foreign Member RS.

Trained as an electrical engineer, he lost both his hands in a lab accident in 1907. He became interested in the theory of higher dimensional algebraic varieties, following in the footsteps of Émile Picard.

He developed the topological tools needed for the purpose and proved some fundamental theorems (later taken up by Hodge and others). In particular he developed intersection theory on manifolds as a basis for algebraic geometry. The Lefschetz fixed point theorem became famous and led to many applications and generalizations, including the Weil conjectures and the Atiyah–Bott theorem for elliptic complexes.

A towering presence in Princeton and great rival of another Solomon (Bochner). Always claimed that his many years of isolation in Kansas had been ideal for his mathematics – no interruptions.

Hermann Weyl

Professor at Göttingen, Zürich and the Institute for Advanced Study, Princeton. Foreign Member RS.

His work on the representation theory of compact Lie groups is well-known

and his 1913 book on Riemann surfaces laid the foundations for all later analytic geometry. He was interested in physics, particularly in the role of symmetry. He introduced gauge theory in an effort to combine the equations of Einstein and Maxwell. He proved rigorously what physicists intuited, that the high frequencies of a quantum system converge to the classical limit.

He was a supporter of Hodge in his work on Harmonic Forms (correcting a technical error), and later of Kodaira, whom he brought from Japan to Princeton. Michael heard him at the 1954 International Congress of Mathematicians in Amsterdam when he presented the Fields Medals to Kodaira and Serre. Michael had the unusual opportunity of writing the

biographical memoir for Weyl (for the US National Academy) 50 years after his death, rectifying an oversight.

2) The French School

Élie Cartan

Professor in Nancy and Paris. Foreign Member RS.

Cartan introduced differential forms and the exterior derivative, extending calculus to manifolds. These are key ingredients in modern differential geometry, and of de Rham cohomology. His work was particularly concerned with the geometric applications of Lie groups, which are manifolds with a group structure. He contributed to the definitive classification of simple Lie groups.

His work had a major influence on mathematical physics.

Father of Henri Cartan.

Henri Cartan

Professor in Strasbourg and Paris. Foreign Member RS.

His contributions were in the global theory that became possible after the pioneering work of Leray. Sheaf theory and cohomology were combined into a powerful tool for the global theory of several complex variables. The purely formal side, developed in conjunction with Eilenberg, gave birth to "homological algebra".

Cartan had many brilliant students at the École Normale Supérieure, notably René Thom and Jean-Pierre Serre, both Fields Medallists. He was also active internationally, becoming the President of the International Mathematical Union. Cartan was the first French mathematician in contact with German colleagues after World War II, in contrast to the ostracism shown after World War I.

Jean Leray

Professor in Paris, Collège de France. Foreign Member RS.

He started off with significant contributions to the Navier–Stokes equations of fluid dynamics. But a quirk of fate diverted him in a different direction.

During the war he was held in a German prisoner of war camp in Silesia. The camp contained many scientists who were allowed to hold seminars. Leray was afraid that, if the Germans realized he was an expert on fluid dynamics, they might put him to military use. So he turned himself into “a useless topologist”. He then developed both spectral sequences and sheaves, the two most powerful techniques of 20th century topology.

Jean-Pierre Serre

Studied at the École Normale Supérieure in Paris under Henri Cartan. Professor at the Collège de France for 38 years, from 1956 on. Foreign Member RS.

He revolutionized both algebraic topology and algebraic geometry by using sheaf theory and spectral sequences, both invented by Leray. He provided techniques to calculate the homotopy groups of spheres and successfully used the Zariski topology as the basis for the cohomology groups of coherent sheaves, following the analytic approach of Henri Cartan.

Serre worked subsequently in number theory and was very influential, with many fruitful ideas and conjectures.

Alexander Grothendieck

Grothendieck started research in functional analysis and solved many hard problems proposed by Dieudonné and Schwartz. He then moved into algebraic

geometry, in the wake of Serre, and transformed the subject completely. He produced a new and elegant proof of the Hirzebruch-Riemann-Roch theorem,

which was purely algebraic. For this he introduced K-theory, which has had an extensive life in other fields. Michael heard his first lectures on the subject in Bonn in 1957 and this motivated the use of K-theory in topology.

His introduction of étale cohomology groups paved the way for the eventual proof by Deligne of the famous Weil conjectures concerning the number of rational points of varieties over finite fields.

An intense, powerful and eccentric personality, he gave up mathematics in 1970, and devoted himself to idealistic causes.

3) The Cambridge School

Paul Dirac

Lucasian Professor of Mathematics at Cambridge 1932–1969. FRS, Honorary FRSE and Nobel Prize.

He discovered how to make quantum mechanics relativistic, by introducing spin and the Dirac operator. He also outlined the programme developed in detail by Feynman and known as the Feynman path integral.

A great believer in the necessity for physical laws to be embodied in beautiful equations, he forecast, in a lecture to the Royal Society of Edinburgh in 1939, that the future of physics would rely on more sophisticated mathematics. Michael attended his Cambridge lectures on quantum mechanics.

A man of few words, his silences were legendary. Despite being a close colleague of Hodge for over thirty years the two do not seem to have had any mathematical exchanges. This left the field between Hodge theory and the Dirac operator open for the next generation (Atiyah–Singer) to exploit.

William Hodge

Undergraduate studies at Edinburgh University and St. John's College, Cambridge. Lecturer at the University of Bristol and subsequently Lowndean Professor of Astronomy and Geometry at Cambridge, 1936-1970. FRS, FRSE.

Hodge's major work, inspired at a distance by Lefschetz, was his theory of harmonic integrals which laid the foundations for future progress in algebraic geometry.

Became Master of Pembroke College, Cambridge and Physical Secretary of the Royal Society. Responsible for bringing the 1958 International Congress of Mathematicians to Edinburgh.

Supervised Michael for his Ph.D. and appointed him as Tutorial Fellow of Pembroke.

John Todd

Part of the large Cambridge school of algebraic geometers under H.F. Baker, he developed the underlying topological invariants and discovered what are now called the Todd polynomials, which play a key part in the Hirzebruch–Riemann–Roch Theorem. Lecturer and Reader at Cambridge 1937–1973. FRS.

Supervised Michael as an undergraduate and oversaw his first publication. Todd was a virtuoso algebraist and put his skills to good use in a joint paper with Michael. Despite teaching Trinity undergraduates for many years Todd was never elected a Fellow of the College, being considered socially gauche. Late in life he became a Fellow of Downing College, where he blossomed.

Abram Besicovitch

Fellow of Trinity College from 1929 until his death. Rouse Ball Professor in Cambridge from 1950 until 1958. FRS.
Early work with Harald Bohr on almost periodic functions. Later he worked on the Kakeya needle problem (now in fashion) and difficult problems of geometric measure theory. Had one student who made his mark, Peter (E.R.) Reifenberg. Taught many Trinity students (including Michael) and left a strong impression because of his eccentric personality and Russian accent. Was interviewed by Michael for the undergraduate magazine *Eureka*.

Periodically "Besi" would introduce Trinity mathematicians to a subtle card game said to be played by Russian peasants, but this tended to endanger Tripos results.

Roger Penrose

Studied at University College, London, then did graduate studies under Todd in Cambridge, as a contemporary of Michael.

Lecturer at Bedford College London 1956-1957. Reader at Birkbeck College London 1964-1973. Rouse Ball Professor at Oxford, 1973-1998. OM, FRS.

Moved from algebraic geometry into theoretical physics. Did important work with Hawking on black holes and subsequently developed twistor theory. This found significant applications in algebraic and differential geometry. As a sideline he produced non-periodic tilings of the plane, which are relevant to quasi-crystals.

Penrose is a prolific author of books on physics and consciousness for the general public.

Simon Donaldson

Pembroke College, Cambridge, graduate student at Oxford under Nigel Hitchin and then Michael.

He applied instanton moduli spaces to make phenomenal discoveries about four-dimensional manifolds, opening up a whole new field which continues to be very fruitful and has deep connections with physics. Donaldson also extended the classical Lefschetz results on pencils in algebraic geometry to a symplectic context.

Fellow of All Souls College, Oxford 1983-1985, thenWallis Professor at Oxford 1985-1998. FRS. Since 1998 Royal Society Research Professor at Imperial College, London.

4) The Russian School

Israel Gelfand

Professor in Moscow and Rutgers. Foreign Member of RS.

His most famous work was in identifying the algebra of continuous functions on a compact space in terms of normed Banach algebras. He then worked on a very wide range of topics in analysis, physics and geometry, notably on the representations of non-compact Lie groups. Henri Cartan compared him with Poincaré.

His Moscow seminar became world famous and his influence on the next generation was unique.

Banned from travelling abroad on the grounds of his secret work, he was eventually permitted to go to Oxford in 1973 to receive an honorary degree. Even so he arrived late and Michael had to interrupt a cricket match to go to Heathrow to collect Gelfand (who received his degree the next day).

Vladimir Arnold

Studied and worked in Moscow, moved to Paris in 1986. Ph.D. with Kolmogorov.

He became famous for the KAM (Kolmogorov–Arnold–Moser) theory of dynamical systems and for his study of the Euler equations of hydrodynamics.

His ideas in symplectic geometry opened up the field and were extremely fruitful. His interests were very broad and his views strong and frequently controversial.

He was a visiting Fellow of Trinity College, Cambridge during the time Michael was Master and he served with him on the Scientific Committee of the European Congress of Mathematics at Barcelona.

Sergey Novikov

Studied and worked in Moscow. Ph.D. with Postnikov. Novikov proved the topological invariance of the Pontrjagin classes, and formulated the far-reaching Novikov conjecture relating characteristic classes and signatures of manifolds to the fundamental group.

Novikov was awarded a Fields Medal at the 1970 International Congress of Mathematicians in Nice. He was not allowed to attend for political reasons, but received the medal when the International Mathematical Union executive committee, chaired by Henri Cartan, and including Michael, met in Moscow.

He later moved into theoretical physics and did important work on periodic integrable systems.

5) Trinity Contemporaries

Frank Adams

Exact contemporary of Michael at Trinity.
Ph.D student of Shaun Wylie. Fellow of Trinity, then Fielden Professor at Manchester (1964–1970) and Lowndean Professor of Astronomy and Geometry at Cambridge (1970–1989). FRS. Leading homotopy theorist of his time. Developed the Adams spectral sequence, and solved outstanding famous problems such as finding the maximum number of independent vector fields on spheres of any dimension, and the Hopf invariant 1 Problem. Adams was killed in a car accident in 1989; the brass memorial plaque to him in the Trinity College Chapel states: *Vir recti iustique tenacissimus aspera montium loca superare nodosque rei topologicae difficiles expedire pari studio pari diligentia gaudebat.* Translation: He was devoted to the truth and to justice. With equal zeal and perseverance he delighted in overcoming the challenges of rock-climbing and in untangling difficult problems in topology.

Ian Macdonald

Undergraduate at Trinity College, Cambridge 1949– 1952, where he was an exact contemporary of Michael Atiyah, James Mackay and Frank Adams. Spent five years in the civil service before returning to mathematics as a lecturer at Manchester University.
Fellow of Magdalen College, Oxford, then Professor at Queen Mary College, London. FRS.
Famous for discovering many remarkable identities which are now understood as extending the theory of Lie groups to the case of the infinite-dimensional loop groups. Wrote influential book on symmetric functions. Co-author with Michael of the popular textbook *Commutative Algebra*.

The lecture notes he took as a student were examples of beautiful calligraphy, in Indian ink on high class paper – irrespective of the lecturer's messy presentation.tangling difficult problems in topology.

T29

James Mackay

Studied mathematics at Edinburgh University 1944–1948, and Trinity College, Cambridge 1950–1952, Lecturer in Mathematics, St.Andrews 1948–1950. FRSE.

Studied Law at Edinburgh, rose rapidly in the legal profession, becoming Dean of the Faculty of Advocates. Lord Advocate and then Lord Chancellor 1987–1997. Chancellor of Heriot-Watt University 1991–2005.

Best man at the wedding of Michael and Lily in Edinburgh in 1955. He gave Michael an Honorary Doctorate at Heriot-Watt, and received the Royal Medal of the Royal Society of Edinburgh, with Michael, in 2003.

6) MFA collaborators

Raoul Bott

Emigrated from Hungary to England 1938, and then to Canada 1940. Graduated in electrical engineering at McGill University 1945. Studied at Carnegie Institute of Technology under J.L. Synge. Professor first at University of Michigan and then at Harvard.

Together with Hans Samelson he applied Morse theory to Lie groups and later discovered his famous periodicity theorems for the homotopy of the classical groups. Subsequently collaborated at length with Michael on index theory, and established the Lefschetz fixed point theorem for elliptic complexes, and (with Patodi) the local index formula.

A larger than life personality, he had many students, including two Fields Medallists (Smale and Quillen).

Friedrich Hirzebruch

Studied in Münster under Heinrich Behnke and in Zürich under Heinz Hopf. Professor at Princeton University and then at University of Bonn. Founder and first director of the Max Planck Institute for Mathematics at Bonn. Foreign Member RS. Honorary FRSE.

Contributed widely to many aspects of geometry, including the theory of Chern classes. He proved the explicit formula for the signature theorem of manifolds given by his L-genus. Famous for his extension of the classical Riemann-Roch theorem. Developed topological K-theory with Michael and made (with Zagier) extensive studies relating topology to number theory.

As initiator of the famous Bonn Arbeitstagung he had a major influence on mathematics in his time. Played a unique role in German mathematics and was a member of the order *Pour le Mérite,* the German equivalent of the Order of Merit.

Vijay Patodi

He received his Ph.D. in 1971 from the University of Bombay under the guidance of M. S. Narasimhan and S. Ramanan at the Tata Institute of Fundamental Research in Bombay.

From 1971 until 1973 Patodi was at the Institute for Advanced Study in Princeton, where he worked with Atiyah, Bott and Singer. Wrote joint papers with all of them and in particular on spectral asymmetry in relation to the index problem for manifolds with boundary.

Patodi returned to the Tata Institute in Bombay but developed serious medical problems which led to his untimely death at the early age of 32. His career had parallels with that of Ramanujan.

Michael Atiyah / Isadore Singer

Worked together on index theory, culminating in the Atiyah–Singer Index Theorem. The work was published in many papers and over several years. It has found unexpected applications in theoretical physics and opened the door to continuing connections between geometry and physics.

Awarded the second Abel Prize, presented by King Harald of Norway (above). The Prize was founded in 2002, and now rivals the Fields Medal, but without the under 40 age restriction.

In 2009 Singer attended Michael's 80th birthday conference in Edinburgh and, a few months later, Michael reciprocated by attending Singer's 85th birthday conference at MIT.

7) Mathematical women

Philippa Fawcett

The daughter of Henry and Millicent Garrett Fawcett, pioneers in women's education. The Fawcett Society is named after her mother.

She studied at Newnham College Cambridge at a time when women were not officially members of the University, but could attend lectures and sit examinations. In the final Mathematical Tripos examinations, where the names were read out in order of merit, the women were slotted in at the appropriate place. In 1890 the name of Phillipa Fawcett was placed above the Senior Wrangler, normally the top position. This became a world-wide sensation with front page photographs in London and New York.

She went on to have a distinguished career in education, as principal assistant to the Director of Education of London County Council.

Grace Chisholm

Grace attended Girton College Cambridge as an undergraduate, and then became a graduate student of Felix Klein in Göttingen. In 1896 she married William Henry Young, who had been her tutor at Girton College.

She worked closely with him on analytic topics, publishing many papers, though some appeared solely under his name so as to improve prospects of getting a job. This was essential since the couple had a full family life, raising six children.

One of their children, Laurence Chisholm Young, became a well-known mathematician and Fellow of Trinity from 1931 until 1935. Michael, as Master, met Young when he returned to Trinity for a Feast at the age of 91. Michael also met his daughter Sylvia Wiegand when he visited the University of Nebraska at Lincoln where Sylvia and her husband were both professors of mathematics.

Dusa McDuff

Dusa McDuff grew up in Edinburgh, the daughter of the famous biologist C.H. Waddington. An undergraduate at Edinburgh, and a graduate student at Cambridge, worked on von Neumann algebras under George Reid. During a year in Moscow she came under the influence of Gelfand.

In the United States since 1977, for many years at Stony Brook and now at Barnard College, part of Columbia University.

Has made extensive and important contributions to symplectic topology. A leading figure supporting women in mathematics. FRS and FRSE.

Harold & Bertha Jeffreys

Harold was Plumian Professor of Astronomy at Cambridge, having made great contributions both to Bayesian probability and geophysics. FRS. Honorary FRSE.

Bertha studied quantum mechanics under Max Born and Werner Heisenberg at Gottingen in 1925-27. She was for many years Director of Studies in Mathematics at Girton College and one of Lily's teachers.

Harold and Bertha wrote the extremely influential universally known simply as textbook *Methods of Mathematical Physics* universally known simply as "J&J".

Harold was a common Cambridge sight on his bicycle until his last years. He always looked the same age (around 60).

Mary Cartwright

First female mathematician to become an FRS.
Student at St. Hugh's College, Oxford 1919–1923. At the time she was only one of five women at Oxford studying mathematics. Oxford Ph.D. 1930. At Cambridge from 1930 onwards.

Honorary FRSE. Sylvester Medal of the Royal Society. Dame of the British Empire.

Mentored by Hardy but famous for her collaboration with Littlewood. Their joint paper, arising from the wartime needs of research on radar, is now recognized as pioneering work on chaos in a dynamical system. She also worked on complex variable theory and supervised Lily for her Ph.D. on "prime ends".

Was Mistress of Girton during Lily's time there.

Michael & Lily Atiyah

Lily: born in Edinburgh 1928, studied at Edinburgh University, Ph.D. under Mary Cartwright at Girton College, Cambridge. Lecturer at Bedford College, London 1954–55. Taught at various Oxford & Cambridge colleges and at Headington School, Oxford.
Michael: born London 1929, went to school at Victoria College, Egypt and Manchester Grammar School. Military service REME. Trinity College Cambridge (Master 1990–97) Ph.D. Under Hodge. Professor at Oxford and the Institute for Advanced Study Princeton. Developed K-theory with Hirzebruch, index theory with Singer and Bott. Contributed to new links between geometry and physics. Fields Medal, Abel Prize. Knight Bachelor, OM, FRS and FRSE. Honorary degrees from Edinburgh and Heriot-Watt. President of the Royal Society 1990–1995. President of the Royal Society of Edinburgh 2005–2008.
Michael and Lily were married in Edinburgh in 1955, with James Mackay as best man.

8) Special

Shiing-Shen Chern / James Simons

Chern: Nankai University, Chicago, Berkeley. Director of MSRI Berkeley, Director of Mathematical Institute Nankai (now named the Chern Institute). Foreign Member RS. Many contributions to differential geometry, but famous for his Chern classes and for his work with Simons.

Simons: studied with Chern and developed together what is now called the Chern-Simons action, much used in physics and at the basis of the knot invariants discovered by Vaughan Jones. Professor at Princeton then at Stony Brook. In 1982 Simons set up the hedge fund investment company Renaissance Technologies, enabling him to become a generous philanthropist through the Simons Foundation. This has supported the Institute for Advanced Study, Princeton, MSRI, IHES and established the Simons Center for Geometry and Physics at Stony Brook. It also supports an extensive fellowship programme.